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Technical throughput of material flow systems required to achieve design flow – determined by simulation with an iteration algorithm

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This article deals with complex material flow systems and series connections of conveyor and operating elements. These can be characterised by a specific availability. The thus resultant overall availability of necessary "technical throughput" of the individual elements for the achievement of a specified throughput. When the conveyor and operating elements are subjected to a stochastic distribution, the interposition of buffers is necessary but these can also lead to a reduction of the necessary throughput due faults. The system behaviour of complex installations can only be investigated by simulation. The parameter changes required in order to achieve specific target values can also be determined by simulation runs in iteration loops.

1. Preamble

In the simplest case material flow systems can be regarded as a series connection of conveyor and operating elements. If the dwell time in each element is Dirac-distributed, and thus constant cycle times prevail, then trivially calculated, the overall throughflow time through the system can be represented by the sum of all individual cycle times and the reciprocal value represents the throughput T .
In reality, the most diverse influences can result in a reduction of this throughput:
Faults can cause an interruption in the material flow until their correction, whereby they can result in standstill of the complete system or can only affect sections when buffers are present.
Stochastically time-distributed faults can be quantitatively registered by the availability ηi of the element. When all availability values of the elements in the series are known, then the overall availability of a system consisting of n elements can be calculated according to

$$\eta_o = \prod_i^n \eta_i$$

Thus in order to ensure a normative throughput T_{req}, the system must achieve this in the uninterrupted operational time and therefore must be designed for a higher technical throughput T_{tech}

$$T_{tech} = \frac{T_{req}}{\eta_o}$$

2. Improving availability through redundancy

So in order to keep this technical throughput low, the overall availability of the system must be as high as possible. If this overall availability is very negatively affected by a single critical element or by a few elements and their individual availability cannot be improved with reasonable outlay, then the availability of these elements can be improved through redundancy.
Cold redundancy is defined as when in the event of a fault a parallel-connected element in standby mode takes over the function of the defective element.
The overall availability of two components in cold redundancy is calculated according to Birolini [Birolini 1997]:

$$\eta = \frac{1+\delta}{1+\delta+\delta^2} \qquad \text{with} \qquad \delta = \frac{MTTR}{MTBF}$$

MTTR … Mean Time To Repair (mean fault duration)
MTBF … Mean Time Between Failures (mean fault-free time)

Warm redundancy is said to exist when in the simplest case in fault-free operation the material flow is divided 1:1 between two elements running in parallel and in the event of failure of an element, the complete throughput is handled by the element remaining in operation. In the determination of the improved availability of two elements switched in redundancy mode consideration must be given to the fact that with warm redundancy there will be different availabilities for the two parallel switched single elements, depending on whether each of the elements are operated at 50% partial load of one element is operated under full load.
In this case the overall availability of the two components in warm redundancy switching mode can be calculated as follows [Birolini 1997]:

$$\eta = \frac{1 + 2\delta_{PL}}{1 + 2\delta_{PL} + 2\delta_{PL}\delta_{FL}} \qquad \text{with} \qquad \delta_{PL} = \frac{MTTR}{MTBF_{PL}} \qquad \delta_{FL} = \frac{MTTR}{MTBF_{FL}}$$

PL ... partial load
FL ... full load

3. Buffer to reduce technical throughput

As already stated, buffers decisively influence the system behaviour, whereby despite their presence the finiteness in their size can lead to total stoppage of the system. In this respect reference should be made to VDI Guideline 3649 [VDI-3649 1992]. This uses an example to demonstrate as to how which buffer capacity in minutes covers which time to repair can be directly read off from an empirically-determined frequency distribution of the times to repair.

Class (min)	Frequency of faults	Time to repair per class (min)
2-<4	56	168
4-<6	13	65
6-<8	7	49
8-<10	9	81
10-<12	0	0
12-<14	0	0
14-<16	7	104
16-<18	0	0
18-<20	3	54
20-<22	0	0
22-<24	2	46
24-<26	0	0
26-<28	1	27
28-<30	0	0
30-<32	0	0
32-<34	0	0
34-<36	0	0
36-<38	1	37
38-<40	0	0
40-<42	0	0
42-<44	0	0
44-<46	0	0
46-<48	1	46
Sum	100	677

Table 1: Classified data of the time to repair [VDI-3649 1992)

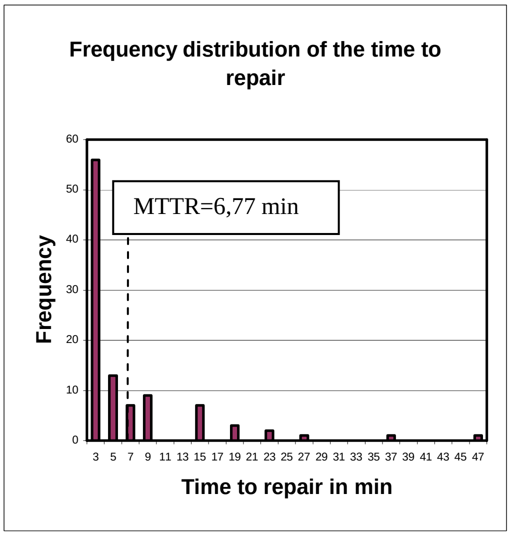
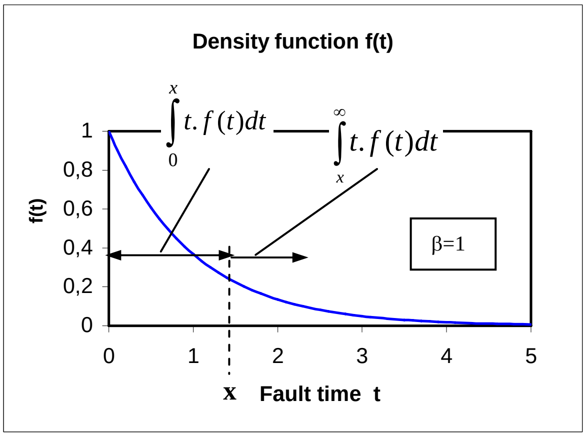


Figure 1: Frequency distribution of the time to repair [VDI-3649 1992]

For example, if a buffer capacity amounting to the MTTR is chosen, then all faults of small MTTRs are fully covered: 168 + 65 = 233 min

In addition, the faults that are greater than MTTR are covered by the time slice of MTTR in each case:
31 . 6.77 = 210 min

Thus of all faults (677 min) 443 min are covered by the buffer. So in summing-up it can be determined that with a buffer capacity x of the amount MTTR and a typical frequency distribution according to Figure 1, which also occurs often in practice, approximately 2/3 of all faults can be covered by the buffer.
In [Decker 2006], this correlation is derived for a very typical continuous distribution function of times to repair, namely the exponential distribution:

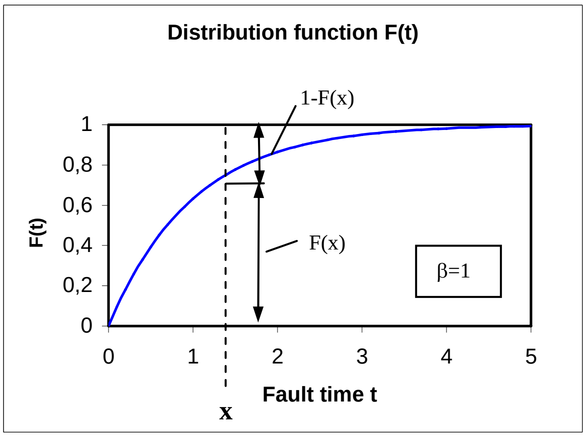


The density function f(t) of the exponential distribution with the constant parameter β>0 is defined for t≥0 as follows:

$$f(t) = \beta . e^{-\beta . t}$$

$$\beta = \frac{1}{E(t)} = \frac{1}{MTTR}$$

Figure 1: Density function f(t) of the exponential distribution [Decker 2006]



The distribution function F(t) of the exponential distribution with the parameter β>0 is defined for t≥0 as follows:

$$F(t) = 1 - e^{-\beta . t}$$

Figure 2: Distribution function F(t) of the exponential function [Decker 2006]

If the time interval (expected value E(x)) that is bridged with a specific buffer capacity x in minutes is calculated, then the bridging proportion f is calculated as follows:

The expected value E(t) of the time to repair can be determined from the density function f(t) in the following manner:

$$E(t) = \int_0^\infty t.f(t)dt = \frac{1}{\beta}$$

Taking into account the two intervals in Figure 2 this results in:

$$E(t) = \int_0^x t.f(t)dt + \int_x^\infty t.f(t)dt$$

Thus the average time period can be bridged with a buffer capacity x can be calculated with the following deliberation: all times to repair that are shorter than x will be bridged. Of faults that last longer then x, the proportion x will be bridged in each case:

$$E(x) = \int_0^x t.f(t)dt + \int_x^\infty x.f(t)dt$$

If the second integral is replaced by the distribution function F(x), this results in:

$$E(x) = \int_0^x t.f(t)dt + x(1 - F(x))$$

$$E(x) = \beta.\int_0^x e^{-\beta.t}.t\ dt + x.e^{-\beta.x}$$

The integral can be solved by means of partial integration and results in the following expression:

$$E(x) = -x.e^{-\beta.x} - \frac{1}{\beta}.e^{-\beta.x} + \frac{1}{\beta} + x.e^{-\beta.x} = \frac{1}{\beta}.\left(1 - e^{-\beta.x}\right)$$

For the bridging proportion f this results in:

$$f = \frac{E(x)}{E(t)} = \beta.E(x) = 1 - e^{-\beta.x} = F(x)$$

$$\boxed{f = 1 - e^{-\beta.x} = F(x)}$$

x...Buffer capacity in minutes
β...Repair rate (=1/MTTR)
F(x)...Function value for a time to repair x

If, for example, a buffer capacity x of the value MTTR chosen, namely x=1/β, then some 63% (f=0.63) of all faults will be bridged. This roughly corresponds to the value 2/3 with the previously considered empirical distribution function in accordance with [VDI-3649 1992]. The frequency distribution represented in Figure 1 actually corresponds approximately to an exponential distribution.

4. Analytical determination of the necessary technical throughput

In accordance with [VDI-3649 1992] as well as [Decker 2006] the necessary technical throughput of the plant components can be determined analytically as follows:

A buffer that is arranged between two plant parts has two functions. It assimilates parts when the subsequent elements are faulty and it releases parts when upstream elements stand still. Therefore for the calculation of the necessary technical throughput the buffer is attributed once to the upstream plant part (Plant Part I) and once to the downstream plant part (Plant Part II).

Buffer allocation to Plant Part I (upstream buffer for Plant Part II, initial situation: buffer full)

If a breakdown of the material flow occurs in Plant Part I, then the elements of Plant Part II can continue working until the buffer is empty. In this way the buffer provides for an increase in the availability of Plant Part I.

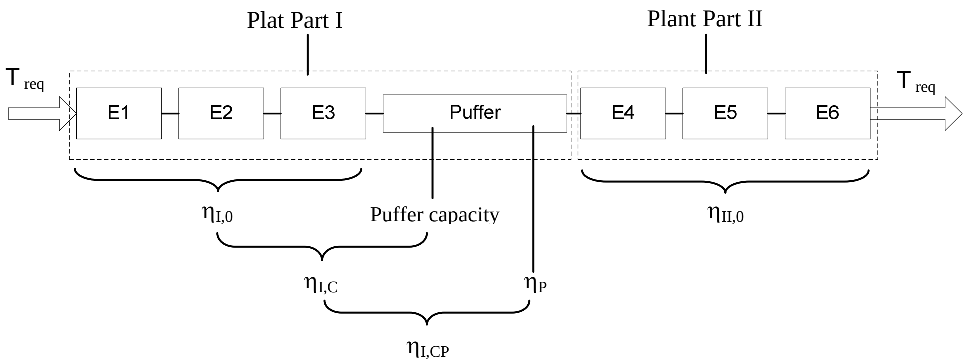


Figure 3: Series connection with buffer (allocation to Plant Part I)

The overall availabilities of Plant Parts I and II without buffer amount to:

$$\eta_{I,0} = \prod_{i=1}^{p} \eta_i \qquad \eta_{II,0} = \prod_{i=p+1}^{o} \eta_i$$

p...number of elements before the buffer

o...number of all elements

The availability of Plant Part I without buffer can also be expressed as follows:

$$\eta_{I,0} = \frac{T_B - T_T}{T_B} = 1 - \frac{T_T}{T_B}$$

T_B...Busy time
T_T...Technical downtime

A proportion f of the times to repair in Plant Part I are covered by the buffer, reducing the time to repair T_T:

$$\eta_{I,C} = \frac{T_B - (T_T - f.T_T)}{T_B} = \frac{T_B - T_T}{T_B} + f.\frac{T_T}{T_B}$$

Therefore the availability of the Plant Part I with consideration of the buffer capacity is given by:

$$\eta_{I,C} = \eta_{I,0} + f(1 - \eta_{I,0})$$

f...proportion of the time to repair that can be bridged by a specific buffer capacity.

If the availability of the buffer should also be taken into account, then this value must be multiplied by η_P:

$$\eta_{I,CP} = \eta_{I,C}.\eta_P$$

The necessary technical throughput then amounts to:

$$T_{II,tech} = \frac{T_{req}}{\eta_{I,CP}.\eta_{II,0}}$$

The buffer capacity can be calculated according to the following formula:

$$C_{II} = x.T_{II,tech} = x.\frac{T_{req}}{[\eta_{I,0} + f(1 - \eta_{I,0})].\eta_P \cdot \eta_{II,0}}$$

x...buffer capacity in time units

Buffer allocation to Plant Part II (downstream buffer for Plant Part I, initial situation: buffer empty)

If a breakdown of material flow occurs in Pant Part II, then the elements of Plant Part I can still continue to function until the buffer is full. In this way the buffer provides for an increase in the availability of Plant Part II.

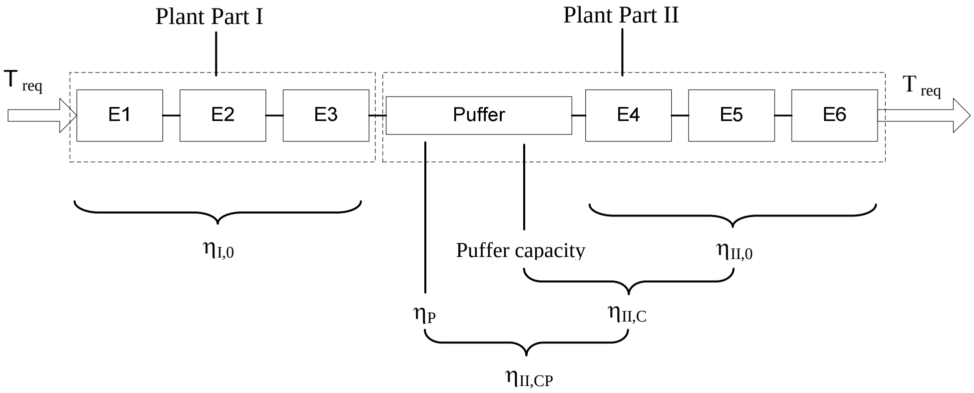


Figure 4: Series connection with buffer (allocation to Plant Part II)

In this case the availability of Plant Part II with consideration of the buffer capacity is given by:

$$\eta_{II,C} = \eta_{II,0} + f(1-\eta_{II,0})$$

Considering the buffer availability, this results in:

$$\eta_{II,CP} = \eta_{II,C} . \eta_P$$

Consequently the necessary technical throughput amounts to:

$$T_{I,tech} = \frac{T_{req}}{\eta_{I,0} . \eta_{II,CP}}$$

The buffer capacity can be calculated according to the following formula:

$$C_I = x . T_{I,tech} = x . \frac{T_{req}}{\left[\eta_{II,0} + f(1-\eta_{II,0})\right] \eta_P \cdot \eta_{I,0}}$$

If a double-acting buffer is installed, then the following buffer capacity is necessary:

$$C = C_I + C_{II}$$

In initial status, C_I must be empty and C_{II} full.

If a buffer with adequate capacity is installed so that all faults can be covered, this leads to a complete decoupling of the two plant parts. Thus when f = 1, the necessary technical throughput amounts to:

$$T_{I,tech} = \frac{T_{req}}{\eta_{I,0} . \eta_P} \qquad T_{II,tech} = \frac{T_{req}}{\eta_{II,0} . \eta_P}$$

But buffers not only influence system behaviour positively insofar that times to repair can be bridged; they also serve as balancing elements when transport and operating times fluctuate causing the so-called intermediate arrival times from element to element to differ and thus possibly queues to form in front of an element. Naturally the expectancy value of the intermediate arrival times of element i must be greater/equal to the expectancy value at the downstream element (i+1), so that the queue does not grow to infinity by continuance of operation. But even with fulfilment of this condition the finiteness of the buffer as a balancing element will lead to interruptions in the material flow.

5. Study of system behaviour using simulation

If we wish to take all these influences into account in a stochastic distribution of the intermediate arrival times and times to repair in an analytical computation, then we encounter limitations even with relatively simple multi-link systems. Thus studies can only be carried out purposefully with discrete simulation.
In [Decker 2006], with the simulation package ARENA based on the simulation language SIMAN a module specially characterising a material flow system was developed; with this, the modelling of a system can be carried out relatively easily. Thus account can be taken not only of faults in all material flow components and buffers with restricted capacities, but also redundantly implemented means of means of conveyance or operating stations. The complete material flow system is visualised on the screen by a variety of animation elements and extensive online statistics such as degrees of utilisation, blocking times, downtimes, technical availabilities, queue lengths and throughflow times are issued.
In principle, using the simulation results can only be ascertained on the basis of given data that describe the system characteristics; thus in this case particularly a given throughput for a steady-state condition of the system. This means that conversely a desired throughput cannot be stipulated in order to achieve necessary system parameters.

6. Iterative simulation sequences to achieve specific target values

The SIMAN/ARENA software incorporates a macrolanguage described as Visual Basic for Applications (VBA) that facilitates the controlling of simulation experiments for the purpose of an iterative approximation of a desired result. So it is possible to select what appears to be a meaningful parameter of the material flow system and, iteratively controlled, to alter it in order to achieve a target value for the throughput. This enables the optimisation of the overall system with judiciously-chosen parameters in relatively few test runs.
Figure 6 is a schematic representation of how the three software tools ARENA, VBA and MS EXCEL interact during the simulation runs.

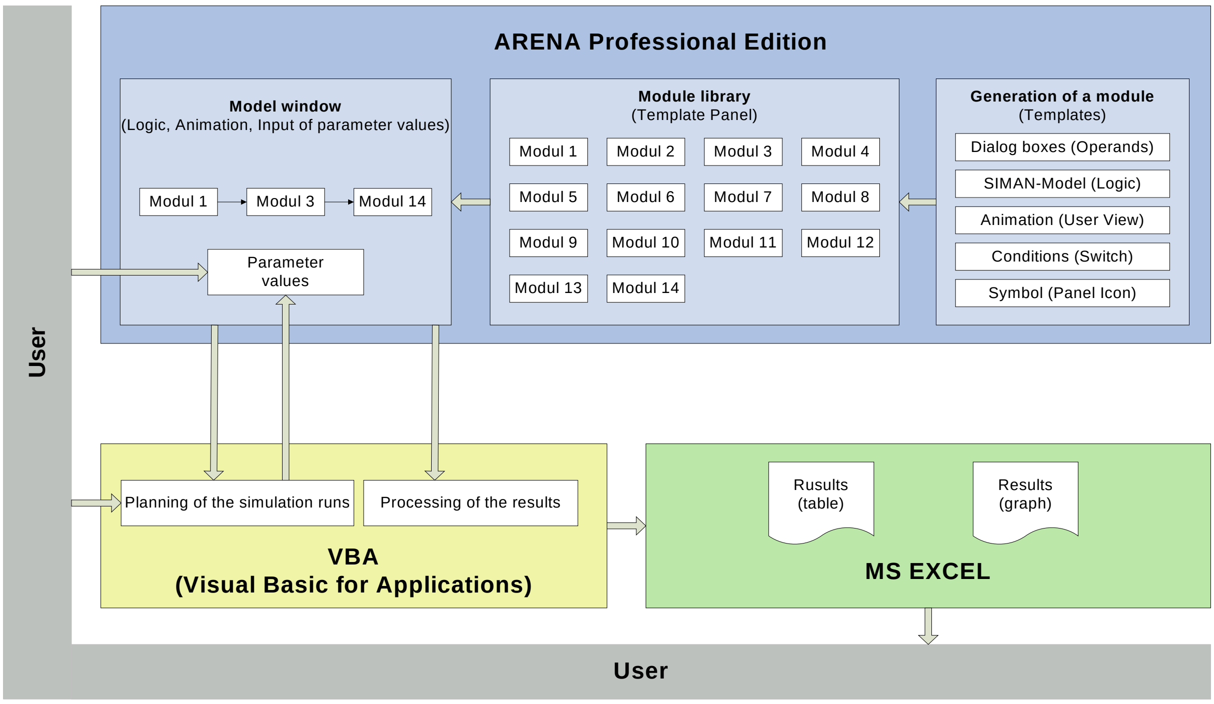


Figure 6: Software structure

The user chooses the appropriate module from the library and places this on the ARENA model window. All relevant parameter values can then be input in clearly-configured dialogue boxes. If the simulation model was fully compiled, the parameters for the iteration routine can be input in a VBA form. The simulation run will then be started. The iteration routine will run through until the value of the result variable lies within the previously defined range. The results are then processed and transferred to MS EXCEL. There the simulation results can be represented either in table or graph form.

The library with the 14 modules is represented in Figure 7.

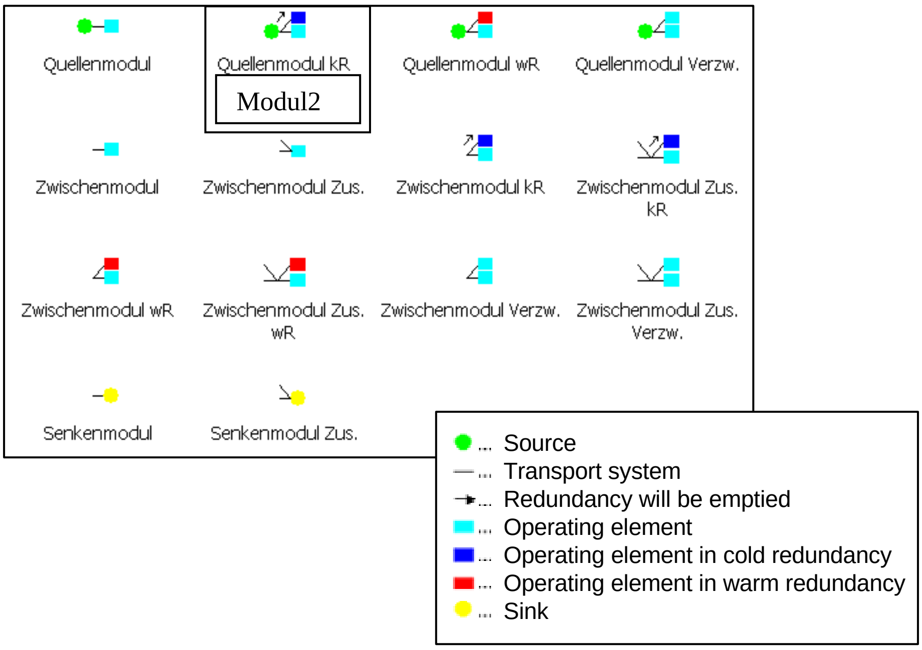
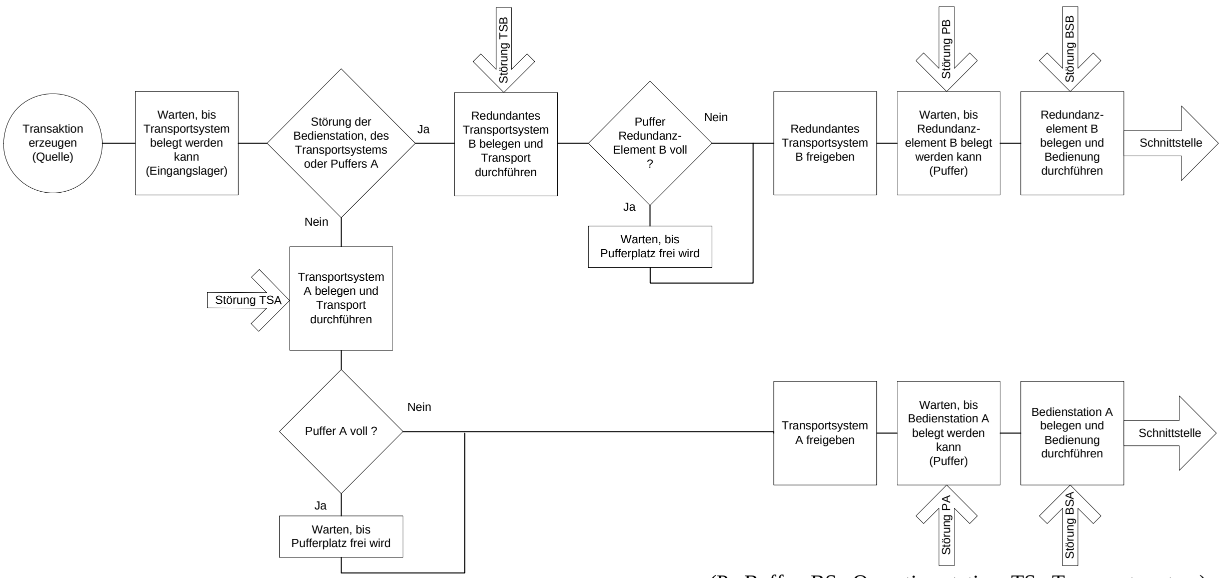


Figure 7: Module library

As am example, Figure 8 describes the model concept for Module 2 (source module with cold redundancy circuit) and the dialogue boxes with the parameters to be input are described in Figure 9. Finally the animation elements also represented on the screen are depicted in Figure 10.



(P...Buffer, BS...Operating station, TS...Transport system)

Figure 8: Model concept for Module 2 (source module with cold recundancy circuit)

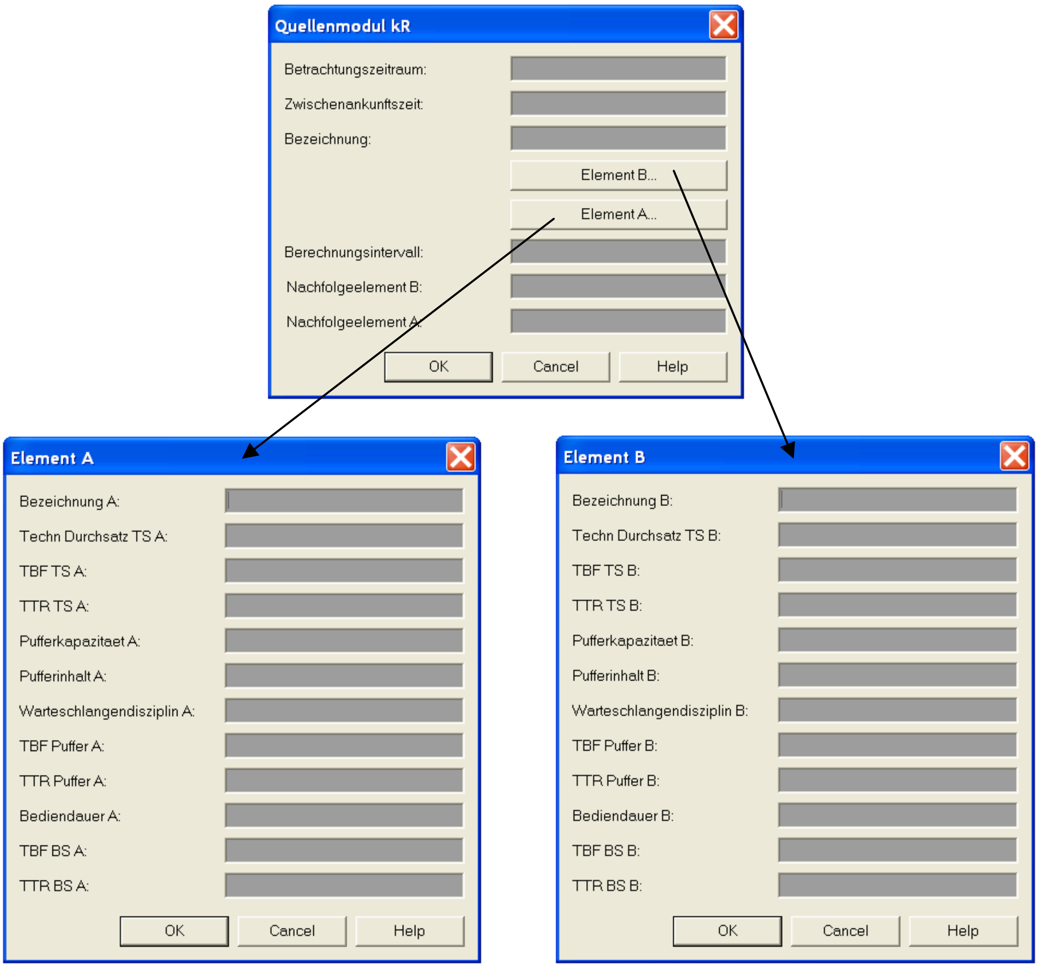


Figure 9: Dialogue boxes for Module 2 (source module with cold redundancy circuit)

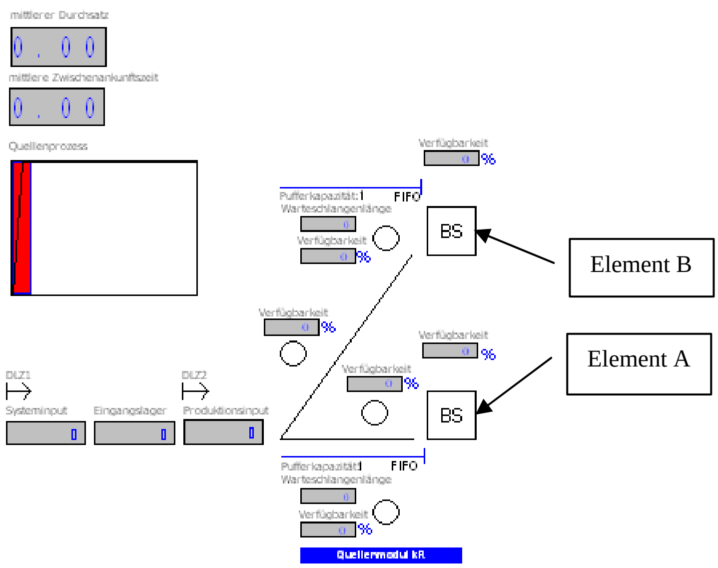


Figure 10: Animation elements for Module 2 (source module with cold redundancy circuit)

For the determination of the necessary technical throughput of material flow elements by means of simulation, with stipulation of the required plant throughput, for example the following procedure can be selected:

1) Determining basic system:

The respective fault behaviour (distribution function) and where appropriate also the type of redundancy circuit must be known for all relevant system components (conveying system, operating stations, buffers). Also, appropriate buffer capacities must be assumed for all buffers.

2) Determining limit values for the transport and service times by means of simple calculation processes:

Limit values (P_{min}, P_{max} in Figure 11) are determined form the transport and service times (with stochastic processes for the appropriate mean values). For example, this can be conducting the calculation once without taking buffers into account and once with the assumption of unlimited buffers.

3) Determining service and transport times for achieving the required throughput by means of iterative parameter variation:

Taking into account the limit values determined under Point 2), all service and transport times are varied. Here the iteration routine runs completely automatically. The principle of iterative parameter variation is illustrated in Figure 11.

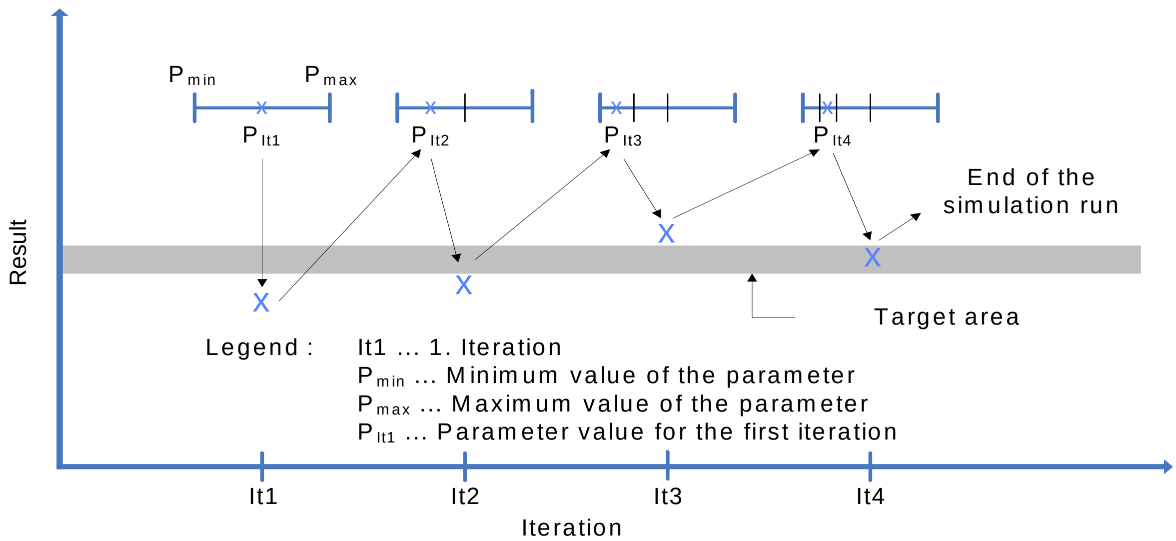


Figure 11: Principle of iterative parameter variation

The 1st iteration starts with the value PIt1=(Pmax+Pmin)/2 (P: service or transport duration). The simulation is then run through and, depending on the result (plant throughput), the new value of the parameter PIt2 is then chosen by halving the corresponding interval. This procedure is repeated until the required specific throughput is achieved. Thanks to this process it is possible with deliberately selected initial values to achieve the desired result after only a few simulation runs.

4) Optimisation of the system parameters

With those material flow components that show very many idle times, the transport or service times can be increased without reducing the desired plant throughput. The extent of the increase can be determined by simulation runs.

5) If necessary, replace system components (simulation modules) and repeat procedure

If the identified transport and service times cannot be achieved in the real plant, changes in the system must be carried out. For example, elements with particularly poor availability can be replaced by other or further redundancy circuits. The iterations can also be conducted with other buffer capacities.

7. Example [Decker 2006]

An example was chosen that could also be solved analytically. It was thus to carry out a validation of the simulation model.

A production system is rigidly linked with an assembly station. The overall system consists of 9 elements:

- 3 conveyor systems (FS1, FS2, FS3);
- 3 buffers (P1, P2, P3);
- 2 production systems in cold redundancy mode (PS1, PS2);
- 1 assembly station (MS).

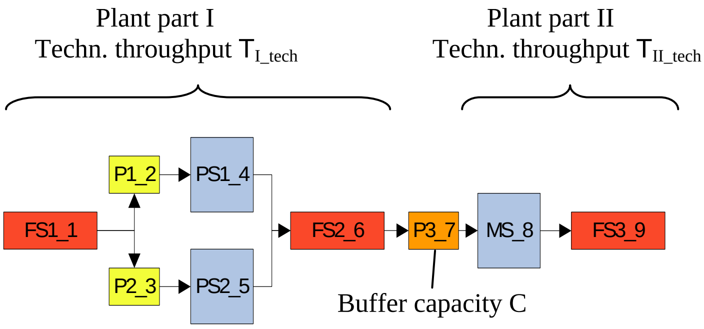


Figure 12: Plant configuration

Buffers P1 and P2 are intended to serve as ready-position stations (Capacity 1). The buffer before the assembly station (P3) should have a limited buffer capacity C. This results in a partial decoupling of the two plant parts I and II.

Both of these elements a certain technical availability is attributed. The fault-free times and the repair times should originate from exponential distributions (see Table 2).

	MTTR in min	MTBF in min	Availability
FS1	15	735	0.98
FS2	15	735	0.98
FS3	30	570	0.95
P1	-	-	1
P2	-	-	1
P3	60	1920	0.97
PS1	20	180	0.9
PS2	20	180	0.9
MS	30	470	0.94

Table 2: Availabilities

The required plant throughput should be 100 units/h. The required technical throughputs for the two plant parts were determined for various buffer capacities both analytically and by means of simulation. [Decker 2006] provides an overview of the calculation possibilities for the necessary technical throughput of plant components.

Shown below by way of example is the calculation of the required technical throughput in Plant Parts I and II for that buffer capacity with which approx. 2/3 of all faults can be bridged (this means that the buffer capacity should be of such a size that an average time to repair of 30 minutes can be bridged, thus x = 30 minutes and f = 0.63):

The overall availability of the two production systems in cold redundancy mode is given by:

$$\eta_{45} = \frac{1+\delta}{1+\delta+\delta^2} = 0.989$$

The overall availability of the Plant Parts I and II is thus:

$$\eta_{I,0} = 0.98 \cdot 0.989 \cdot 0.98 = 0.949$$
$$\eta_{II,0} = 0.94 \cdot 0.95 = 0.893$$

Taking account of the bridging effect of Buffers P3 (f = 0.63) results in the following availabilities:

$$\eta_{I,C} = \eta_{I,0} + f(1-\eta_{I,0}) = 0.981$$
$$\eta_{II,C} = \eta_{II,0} + f(1-\eta_{II,0}) = 0.960$$

As also the buffer itself can be subject to faults, these values must then be multiplied by the availability of Buffer P3:

$$\eta_{I,CP} = \eta_{I,C} \cdot \eta_{P3} = 0.952$$
$$\eta_{II,CP} = \eta_{II,C} \cdot \eta_{P3} = 0.931$$

The required technical throughputs in the Plant Parts I and II with a normative throughput of 100 units/h are thus:

$$T_{I,tech} = \frac{T_{req}}{\eta_{I,0} \cdot \eta_{II,CP}} = 113.2 \ unit/h$$
$$T_{II,tech} = \frac{T_{req}}{\eta_{I,CP} \cdot \eta_{II,0}} = 117.6 \ units/h$$

And the buffer capacity C is thus:

$$C = C_I + C_{II} = x \cdot T_{I,tech} + x \cdot T_{II,tech} = 57 + 59 = 116 \ units$$

Table 3 shows a comparison of the analytical results and the simulation results of the iterative parameter variation for a buffer capacity of 116 units.

	Calculation process	Simulation
$D_{I,tech}$	113.2 units/h	112.6 units/h
$D_{II,tech}$	117.6 units/h	116.2 units/h

Table 3: Comparison of results

Calculations were carried out for various buffer capacities x in minutes. The resultant buffer capacities in units are summarised in the table below.

Buffer capacity x in min	10	20	30	40	50	60
Buffer capacity K in units	40	78	116	153	190	227

Table 4: Buffer capacities of buffer P3

Shown below by way of example in Figure 13 are the results of the required technical throughputs in Plant Part II, namely in the assembly station, compared for the various buffer capacities according to Table 4.

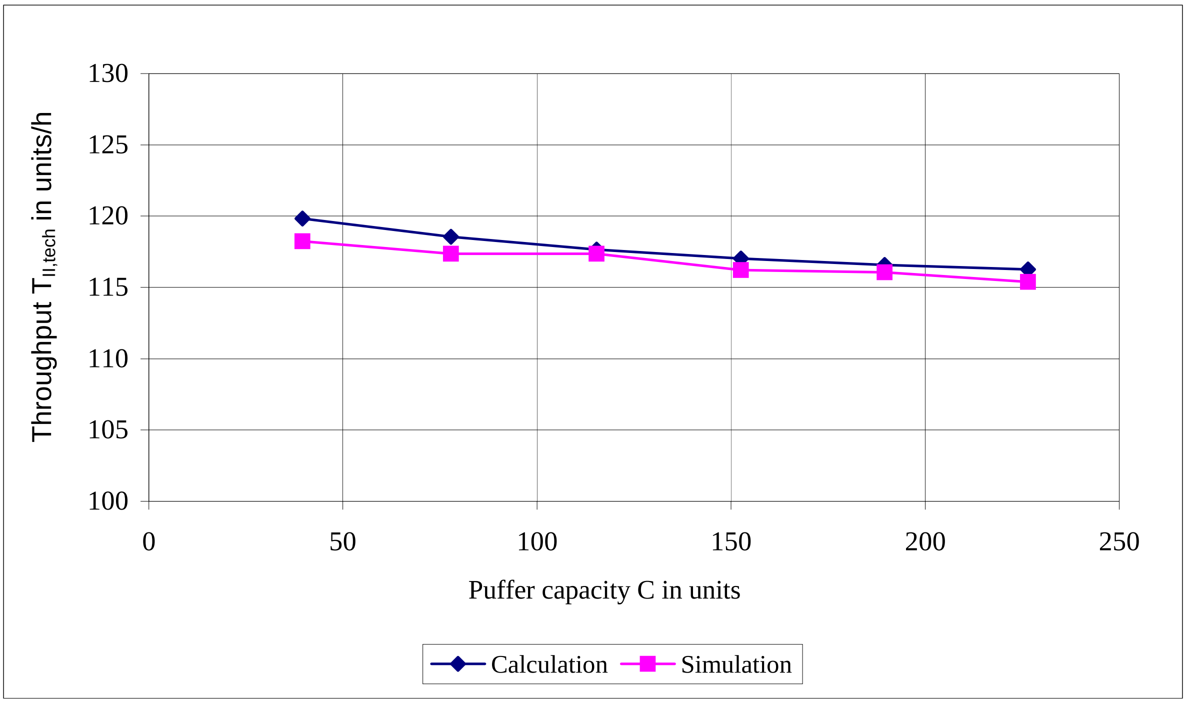


Figure 13: Comparison of results

Literature

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